Activity Sheet for the September, 2014, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. Compute the sum of the first 99 positive integers.

2. A series of figures are created with dots as shown below. After Figure 1, each figure is created by adding a new row of dots that has one more dot than the previously added row of dots. How many dots total are there in Figure 10?
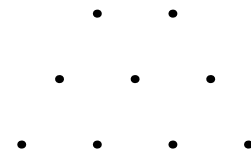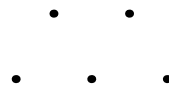
Figure 1   Figure 2   Figure 3   Figure 4

3. Find the missing terms in each of the following arithmetic sequences:

   (a) $5, 11, \_\_\_, \_\_\_, \_\_\_$

   (b) $\_\_\_, \_\_\_, \_\_\_, 5, 11$

   (c) $5, \_\_\_, \_\_\_, \_\_\_, 11$

4. Without writing anything, find the following:

   (a) The largest integer whose square is less than one million.

   (b) The least positive three-digit integer that leaves a remainder of 1 when divided by 7.

   (c) The least positive four-digit integer that leaves a remainder of 5 when divided by 9.

The Problems

First Problem: Consider the pattern of square grids shown. The sum of the numbers in the square at Stage 3 is 27. If the pattern continues, what will be the sum of the numbers in the square grid at Stage 7?

1	

Stage 1

2	3
1	2

Stage 2

3	4	5
2	3	4
1	2	3

Stage 3

4	5	6	7
3	4	5	6
2	3	4	5
1	2	3	4

Stage 4
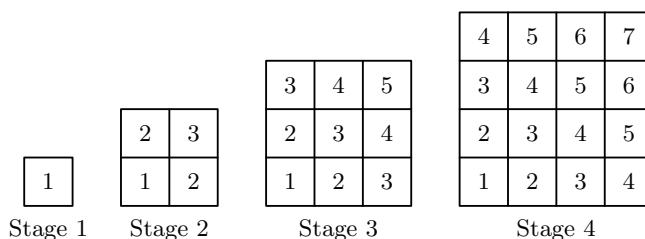
Second Problem: Consider the sequence $1, 3, 4, 7, 11, 18, 29, \ldots$ in which each term is the sum of the two previous terms after the first two terms. How many of the first 100 terms of this sequence are multiples of 5?

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Follow-up Problems

5. If $S(n)$ is a function that returns the sum of the first n positive integers, then what is $S(20) - S(19)$?

6. We draw the same sequence of Figures as in Problem 2, but we connect the dots in each Figure following the pattern shown below. If each segment from a dot to its nearest neighbors has length 1, then what is the total length of all of the segments in Figure 10?

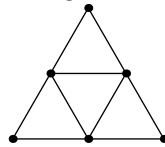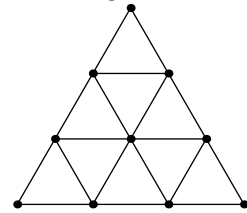Figure 1   Figure 2   Figure 3   Figure 4

7. Jenny has a list of twelve numbers. For each integer n from 1 to 12, the product of the first n numbers in the list is n^2. What is the last number in the list?

8. The sum of the first n terms of a sequence, $a_1 + a_2 + \cdots + a_n$, is given by the formula $S_n = n^2 + 4n + 8$. The sum of the first three terms, for example, is $S_3 = (3)^2 + 4(3) + 8 = 29$. What is the value of a_6?

Wow! Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).