MATHCOUNTS®

Special Right Triangles

Coach instructions: *Give students around 10 minutes to go through the warm-up problems.*

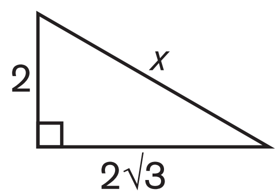
Warm-Up!

Try these problems before watching the lesson.

Note: *The terms in blue italics commonly appear in competition problems. Make sure Mathletes understand their meaning!*

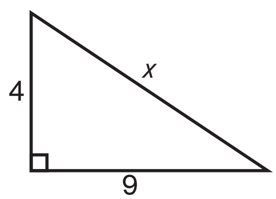
For each of the following problems, determine the value of x in each figure by using the Pythagorean Theorem. Express the value of x as an integer or in ***simplest radical form***, whichever applies to your answer.

1.



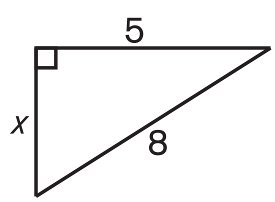
$2^2 + (2\sqrt{3})^2 = x^2$
$4 + 12 = x^2$
$16 = x^2$
$x = 4$

2.



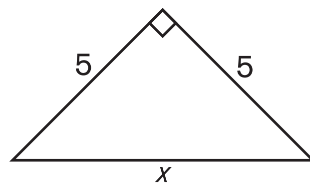
$4^2 + 9^2 = x^2$
$16 + 81 = x^2$
$97 = x^2$
$x = \sqrt{97}$

3.



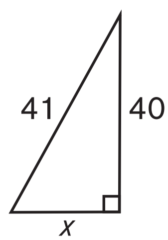
$x^2 + 5^2 = 8^2$
$x^2 + 25 = 64$
$x^2 = 39$
$x = \sqrt{39}$

4.



$5^2 + 5^2 = x^2$
$25 + 25 = x^2$
$50 = x^2$
$x = \sqrt{50} = \sqrt{(2 \cdot 25)} = 5\sqrt{2}$

5.



$x^2 + 40^2 = 41^2$
$x^2 = 41^2 - 40^2$
$x^2 = (41 - 40)(41 + 40)$
$x^2 = (1)(81) = 81$
$x = \sqrt{81} = 9$

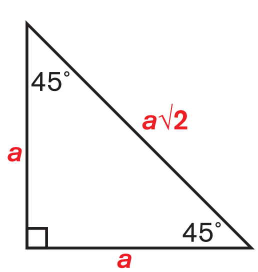
The Problems

Coach instructions: *After students try the warm-up problems, play the video and have them follow along as the ratios between side lengths in special right triangles are shown.*

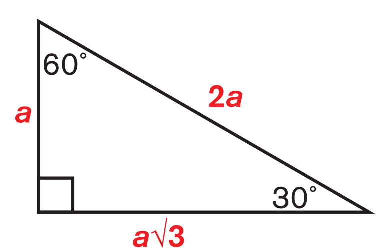
Take a look at the following problems and follow along as they are explained in the video.

Determine the relationship between the side lenghts of the special right triangles below.

6.



7.



Piece It Together

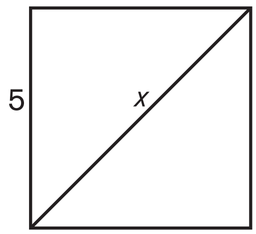
Coach instructions: *After watching the video, give 15 minutes to try the next five problems.*

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

For each of the following problems, determine the value of *x* in each figure by finding the special right triangles. Express the value of *x* as an integer or in simplest radical form, whichever applies to your answer.

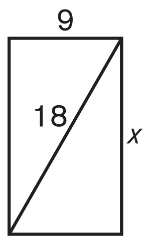
8. square

The diagonal of the square divides it into two 45-45-90 triangles. The sides of the square will be the shorter length in the ratio and the value of *x* will be $5\sqrt{2}$.



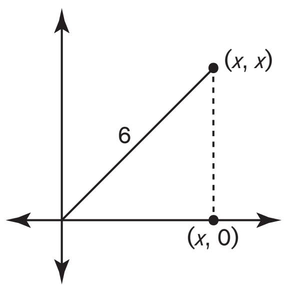
9. rectangle

The diagonal of the rectangle divides it into two right triangles. We that the hypotenuse is twice the length of one of the sides meaning this is a 30-60-90 triangle. The length of *x* will be $9\sqrt{3}$.



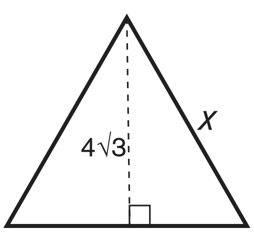
10. coordinate plane

The coordinate point (*x*, *x*) allows us to think of this line as the hypotenuse of a 45-45-90 triangle. We are looking for what would make $x\sqrt{2} = 6$. We know that $6 = 3 \cdot 2$ and $2 = \sqrt{2} \cdot \sqrt{2}$, so $x = 3\sqrt{2}$.



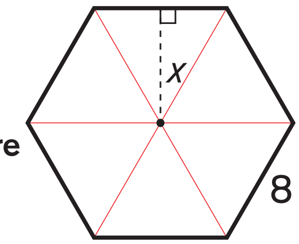
11. *equilateral triangle*

From the video we know that an equilateral triangle can be split down the middle into two 30-60-90 triangles. This means $4\sqrt{3}$ is our longer side length. The shorter side will be 4 and the hypotenuse, *x*, will be 8.



12. *regular hexagon*

A regular hexagon has six sides of equal length. We can divide the figure into six equilateral triangles with side length 8. Using the 30-60-90 ratios we find that $x = 4\sqrt{3}$.

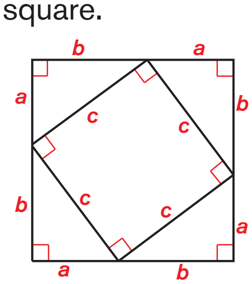


Coach instructions: *Once your students have completed the problems, let them try this proof. You might want to give them hints to help them along.*

Optional Extension

To extend your understanding and have a little fun with math, try the following activities.

In the warm-up you practiced using the Pythagorean Theorem to solve for side lengths in right triangles. In the video, the Pythagorean Theorem was used to show the side length ratios in special right triangles. But how do we know the Pythagorean Theorem is true? There are 367 unique proofs of this theorem. For this extension, try to uncover one known proof. The figure below gives a hint. The figure shows a smaller square inscribed inside a larger square.



Assign variable side lengths a, b and c to the figure as shown. We can think of this problem as solving for the area of the inner square by subtracting the area of the four triangles from the larger square. Setting up this equation we get:

$$(a + b)^2 - 4(\tfrac{1}{2}ab) = c^2$$

$$a^2 + 2ab + b^2 - 2ab = c^2$$

$$a^2 + b^2 = c^2$$