MATHCOUNTS® Minis September 2009 Activity Solutions

Warm-Up!

1. On a fair six-sided die, there is one of each of the numbers 1 through 6. So, the probability of rolling a 3 is **1/6**.

2. Each die has a number 1 through 6 on each of its sides, which means we need to find how many combinations of these numbers can sum to 4: (3, 1), (1, 3), (2, 2). So, there are 3 ways to get a sum of 4 when rolling 2 dice. There are 6 × 6 = 36 possible ways to roll two dice, so the probability of rolling a sum of 4 is 3/36 = **1/12**. We could get a sum of 10 by rolling (4, 6), (6, 4), (5, 5), so there are also 3 ways to get a sum of 10 when rolling 2 dice. Thus, the probability is also 3/36 = **1/12**.

3. If there are three dice rolled, the minimum possible sum of the numbers rolled is 1 + 1 + 1 = 3, and the maximum possible sum is 6 + 6 + 6 = 18. Thus, the possible values of the sum of the numbers rolled are **anything between 3 and 18, inclusive**.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

4. If two dice are rolled, there are 5 ways to get a resulting sum of 6: (1, 5), (5, 1), (2, 4), (4, 2), (3, 3). There are 6 × 6 = 36 possible ways to roll two dice, so the probability of rolling a sum of 6 is **5/36**.

5. With three dice, there are several ways to obtain a sum of 7. Take 1 + 1 + 5 = 7. There are 3 ways we could roll these three numbers: (1, 1, 5), (1, 5, 1), (5, 1, 1). Next, we can look at 1 + 2 + 4 = 7. Since all three values are different in this case, there are 3! = 3 × 2 × 1 = 6 ways we could roll these three numbers. For 1 + 3 + 3 = 7, there are 3 possible rolls: (1, 3, 3), (3, 1, 3), (3, 3, 1). Finally, for 2 + 2 + 3 = 7, there are 3 possible rolls: (2, 2, 3), (2, 3, 2), (3, 2, 2). Thus, there are 3 + 6 + 3 + 3 = 15 ways to roll a sum of 7 with three dice. There are 6 × 6 × 6 = 216 possible outcomes when rolling three dice, so the probability of rolling a sum of 7 is **15/216**.

6. With three dice, there are several ways to obtain a sum of 14. Take 6 + 6 + 2 = 14. There are 3 ways we could roll these three numbers: (6, 6, 2), (6, 2, 6), (2, 6, 6). Next, we can look at 6 + 5 + 3 = 14. Since all three values are different in this case, there are 3! = 3 × 2 × 1 = 6 ways we could roll these three numbers. For 6 + 4 + 4 = 14, there are 3 possible rolls: (6, 4, 4), (4, 6, 4), (4, 4, 6). Finally, for 5 + 5 + 4 = 14, there are 3 possible rolls: (5, 5, 4), (5, 4, 5), (4, 5, 5). Thus, there are 3 + 6 + 3 + 3 = 15 ways to roll a sum of 14 with three dice. There are 6 × 6 × 6 = 216 possible outcomes when rolling three dice, so the probability of rolling a sum of 14 is **15/216**.

7. When rolling three dice, a sum of 7 is the minimum sum of 3 plus 4. On the other hand, 14 is the maximum sum of 18 minus 4. So, like in problem 2, the cases are symmetric. Once you have solved problem 5, you also know the answer to problem 6.

8. In Pascal's Triangle, we see $11^3 = 1331$ in the third row. We see $11^2 = 121$ in the second row and $11^4 = 14,641$ in the fourth row. This is one of the many patterns that continues throughout Pascal's Triangle. However, when you get to row 5 and beyond (where two-digit numbers come into play), you'll need to do some carrying over.

9. G. H. Hardy was an English mathematician, who mentored an Indian mathematician named Srinivasa Ramanujan. Hardy once took taxi #1729 to visit Ramanujan, who discovered that $1729 = 10^3 + 9^3 = 12^3 + 1^3$ (so, $12^3 = 1728$). This number, 1729, is the only number that can be expressed as the sum of the cubes of *two* different sets of numbers. Today, "taxicab numbers" is the term used to describe the smallest integer that can be expressed as a sum of two positive integer cubes in n distinct ways (so, 1729 is the 2nd taxicab number).