MATHCOUNTS®

Ratios & Simple Probability

Warm-Up!

Try these problems before watching the lesson.

Coach instructions: *Give students around 10 minutes (2 minutes per problem) to go through the warm-up problems.*

Note: *The terms in blue italics commonly appear in competition problems. Make sure Mathletes understand their meaning!*

1. What is $\frac{9}{30}$ as a ***common fraction***?

   MATHCOUNTS problems often ask you to express your answer as a common fraction, or a fraction reduced to its simplest form. For 9/30, both the numerator and denominator can be divided by 3 to give us 3/10 which is a common fraction since neither the numerator or denominator share a common factor other than 1.

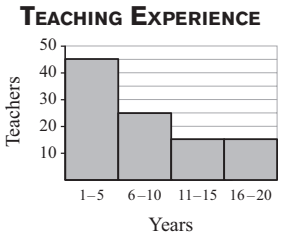
2. In a pasture there are 12 white horses and the rest are black. If there are 52 horses in the pasture what fraction are black? Express your answer as a ***common fraction***.

   If 12 of the 52 horses are white, then 52 − 12 = 40 horses are black. This means 40/52 or 10/13 of the horses are black.

3. Lakeland Middle School's MATHCOUNTS club has 14 girls and 12 boys. What fraction of the members are girls? Express your answer as a common fraction.

   If the club has 14 girls and 12 boys, then there are 14 + 12 = 26 team members. Of the 26, 14/26 = 7/13 are girls.

4. The graph shows the results when 100 teachers reported the number of years they have been teaching. What percentage of these teachers have been teaching more than 5 years but no more than 15 years?



   From the graph, we see that 25 teachers have been teaching 6-10 years and 15 teachers have been teaching 11-15 years. This means that of the 100 teachers 25 + 15 = 40 have been teaching more than 5 years but less that 15 or 40/100 = 40%.

5. There are three times the number of orange fish as blue fish in a tank at the pet store, and there are no other fish. What percentage of the fish are orange?

   We don't know exactly how many fish are in the tank, but we do know that for every 1 blue fish there are 3 orange fish. This gives us a ratio of 3/4 or 75%.

The Problems

Coach instructions: *After students try the warm-up problems, play the video and have them follow along with the solutions.*

Take a look at the following problems and follow along as they are explained in the video.

6. A drawer contains five socks: two green and three blue. What is the probability that a sock pulled out of the drawer at random will be green? Express your answer as a ***common fraction***.

   Solution in video. Answer: 2/5.

7. Nine cards are numbered 1 through 9. What is the probability of selecting a card with a number greater than four or an even number? Express your answer as a ***common fraction***.

   Solution in video. Answer: 7/9.

8. What is the probability that a randomly selected positive integer less than or equal to 3000 is a multiple of 5? Express your answer as a ***common fraction***.

   Solution in video. Answer: 1/5.

Piece It Together

Coach instructions: *After watching the video, give students 10 to 15 minutes to try the next five problems.*

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

9. What is the probability that Kai will select a red or blue marble when he selects one marble from a jar containing three green, two red and five blue marbles? Express your answer as a ***common fraction***.

   Our total number of possible outcomes is the total number of marbles or 3 + 2 + 5 = 10. The number of favorable outcomes is 2 + 5 = 7 since Kai can draw a red or a blue. The probability is therefore 7/10.

10. The table shows the number of students enrolled for each grade at East-West High School. Every 9th- and 10th-grade student at East-West is automatically entered into a drawig to win a new tablet computer. One winning students will be chosen randomly. What is the probability that a 10th-grader at East-West will win the drawing? Express your answer as a ***common fraction***.

Grade	Enrollment
9	192
10	136
11	129
12	93

   Our total number of possible outcomes will be the number of 9th- and 10th-grade students since they are the only ones in the drawing, and our number of favorable outcomes will be the number of 10th-grade students. This gives us a probability of 136/(192 + 136) = 136/328 = 17/41.

11. What is the probability that a positive integer less than or equal to 24 is a multiple of 4? Express your answer as a ***common fraction***.

Similar to the third problem in the video, you might notice that 24 is a multiple of 4. One in every four consecutive integers will be a multiple of 4 so the probability is 1/4.

12. A jar contains 28 marbles. Half of the marbles are red. Half of the non-red marbles are white and the rest are blue. Todd chose a white marble at random and kept it. What is the probability that when Hosea now draws a marble it will also be white? Express your answer as a ***common fraction***.

Of the 28 marbles, 1/2 × 28 = 14 are red. Of the remaining 14, 1/2 × 14 = 7 are white and the remaining 7 are blue. If Todd chose a white marble, the total number of marbles is now 27 with 14 red, 7 − 1 = 6 white and 7 blue. The probability that Hosea will draw a white marble is 6/27 = 2/9.

Optional Extension

Coach instructions: *Once your students have completed the problems and feel they have a comfortable understanding of the concept, let them play this dice game. If you don't have dice, you can use random number generator on the calculator.*

To extend your understanding and have a little fun with math, try the following activities.

Practice simple probability calculations by playing a dice game! Get a six-sided die and a partner. Decide who will be Player 1 and who will be Player 2 for round one (you will switch off after every round). Player 1 starts the round by rolling the six-sided die. Player 2 now has two options: roll or don't roll. If Player 2 chooses not to roll, she gets 0 points and Player 1 gets +1 point. If Player 2 chooses to roll, there are three possible outcomes: she rolls a number lower than Player 1, she rolls the same number as Player 1 or she rolls a number higher than Player 1. If she rolls a number lower, then Player 1 gets +1 point and she gets −1 point. If she rolls a number equal to Player 1, they both get 0 points. If she rolls a number higher, then Player 1 gets 0 points and she gets +1 point. These four outcomes are summarized in the table below.

	No roll	Player 1 > Player 2	Player 1 = Player 2	Player 1 < Player 2
Player 1	+1	+1	0	0
Player 2	0	−1	0	+1

Switch off who is Player 1 each round. Play 10 rounds and keep a running score. The winner is the one with the most points at the end of the rounds. If you are tied, continue playing until someone pulls ahead in points. Make sure you are considering the probability each time you choose to roll or not roll!

After Mathletes finish their problems, they can play this game to keep practicing simple probability calculations. Every roll and every new round presents a scenario where they will have to judge the probability of scoring and weigh it with the risk of losing points.

Here is an example scenario Mathletes might encounter:

Player 1 rolls a 4. There is a 3/6 = 1/2 chance Player 2 will roll a lower number, a 1/6 chance they will rolls the same number and a 2/6 = 1/3 chance they will roll a higher number. Looking at this, Player 2 may see they are most likely to roll a lower number and therefore may choose to pass on the roll.

Note: *If your Mathletes enjoy playing this game, then check out the National Math Club. It is free to register for on our website and will give you access to even more math concept games for you to use!*