*This practice plan was created by **Tyler Erb**, a math teacher and coach at Community House Middle School. Tyler created numerous free resources for MATHCOUNTS coaches in his role as the 2021-2022 DoD STEM Ambassador for MATHCOUNTS. Find more resources and information at **dodstem.us**.*

Modular Arithmetic

Warm-Up!

Try these problems before watching the lesson.

1. It is currently 4:30 p.m. What time will it be in 2022 minutes?

2. It is currently 4:30 p.m. What time will it be in 2022 hours?

3. What is the units digit of 4^{2022}?

4. What is the remainder when 25^{2022} is divided by 6?

5. You have a pile of marbles. When you divide them up into piles of 7, you have 1 left over. When you divide them up into piles of 4, you have 2 marbles left over. If you know you have more than 111 marbles, what is the least number of marbles you can have?

The Problems

Take a look at the following problems and follow along as they are explained in the video.

6. What is the remainder of the 2022nd term in the Fibonacci sequence divided by 3?

7. What is the smallest positive value of $x + y$ if $43x + 85y = 2022$ and both x and y are integers?

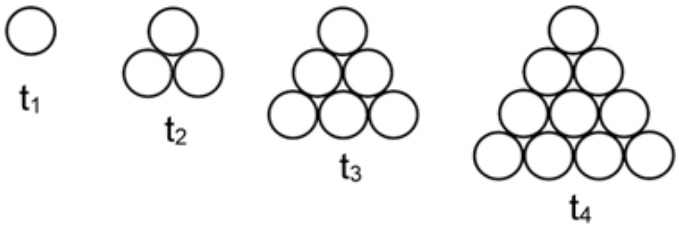
8. What is the units digit of 2022^{2022}?

9. What are the last two digits of 504^{4044}?

Piece It Together

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

10. A triangular number, t_n, is a number that is created by forming an equilateral triangular grid of points where the first row is 1, and each subsequent row contains one more element than the previous one. The first four terms are shown and are 1, 3, 6, 10. What is the remainder of the 2022nd number when it is divided by 7?



11. What is the units digit of the remainder when 16^{2022} is divided by 35?

12. What is the units digit of the remainder when 144^{2345} is divided by 35?

13. What are the last two digits of the product of 34^{2021} and 37^{2022} divided by 35?

14. You and a group of 3 friends find a treasure chest filled with gold coins. You find that if you split the coins up into piles of 4 or 7, there are 2 remaining. If you split the coins into stacks of 9, you find that there is a remainder of 1. You know that there are at least 100 coins. What is the least amount of gold coins possible?

15. There are exactly three ordered pairs of positive integers (x, y) that satisfy the equation $12x + 21y = 261$. What is the sum of all the x coordinates of these ordered pairs?

16. There is exactly one ordered pair of positive integers (x, y) that satisfies the equation $76x + 39y = 881$. What is the sum of x and y?

Optional Extension

To extend your understanding and have a little fun with math, try the following activity.

A mogul has decided to buy up land and make a new development with 108 acres of land. The developer wants to have plots that are 2 acres, 5 acres or 7 acres. There must be exactly 20 total plots, and there must be at least one plot of each size. How many total combinations of plots are there?