MATHCOUNTS®

Distance = Rate × Time

Coach instructions: *Give students around 10 minutes (2 minutes per problem) to go through the warm-up problems.*

Warm-Up!

Try these problems before watching the lesson.

1. In gym class, Travis ran 95,040 inches. Mauricio ran 1.8 miles. Alex ran 11,088 feet. Who ran the farthest, given 1 mile = 5,280 feet?

   We'll need to convert each boy's distance so that they are all in the same units. Converting miles to inches takes 2 steps (converting miles to feet, then feet to inches), and vice versa. But, converting each distance to feet requires only one step, so let's convert all distances to feet. If we convert Travis's distance to feet, we find that he ran 95,040/12 = 7,920 feet. If we convert Mauricio's distance to feet, as well, we find that he ran 1.8 × 5,280 = 9,504 feet. We already know that Alex ran 11,088 feet. So, we can see that Alex ran the farthest.

2. Zola the ant walked 1 meter, then 15 cm and then 3.7 cm. How many millimeters did Zola walk?

   We're asked to find the number of millimeters Zola walked, so we'll need to convert each distance to millimeters. We know that 1 m = 100 cm = 1,000 mm. So, if Zola walked 1 m, she walked 1,000 mm. Knowing that 100cm = 1,000mm, we can see that there are 1,000/100 = 10 mm per cm. So, when Zola walked 15 cm, she walked 15 × 10 = 150 mm. Finally, when Zola walked 3.7 cm, she walked 3.7 × 10 = 37 mm. Taking all these distances together, we get that Zola walked 1,000 + 150 + 37 = 1,187 millimeters.

3. Blake traveled 117 mi in 2.25 hours to come home from college. What was the average speed in miles per hour at which Blake traveled?

   We can set up a proportion to solve. We know that Blake traveled 117 miles in 2.25 hours, and we want to know how many miles he could travel in 1 hour to get his average speed in miles per hour. So, 117/2.25 = x/1. Cross multiply to get 117 = 2.25x. Dividing by 2.25 on both sides of the equation gives us x = 52 miles per hour. Alternatively, since rate = distance / time, we can take 117 miles divided by 2.25 hours to get 52 miles per hour.

4. Mazen takes 8 minutes to bike 2 miles to work. What is his average speed, in miles per hour?

   Since we're asked about speed in miles per hour, let's first convert our time units into hours. Knowing there are 60 minutes in 1 hour, 8/60 = 2/15 hours. If Mazen travels 2 miles in 2/15 hours, we want to know how many miles she can travel in 1 hour to get her average speed in miles per hour. Again, we can set up a proportion to solve. So, 2/(2/15) = x/1. Cross mulitply to get 2 = (2/15)x. Dividing by 2/15 on both sides of the equation gives us x = 15 miles per hour. Alternatively, since rate = distance / time, we can take 2 miles divided by 2/15 hours to get 15 miles per hour.

5. A speed of 60 miles per hour is equal to 88 feet per second. If the speed limit in a school zone is 15 miles per hour, what is the speed limit in this zone in feet per second?

The school zone speed limit of 15 mi/h is one-quarter of 60 mi/h = 88 ft/s. So, 15 mi/h is equivalent to 88/4 = 22 ft/s. Alternatively, we can solve by doing the necessary unit conversions. There are 60 seconds in 1 minute and 60 minutes in 1 hour. So, there are 60 × 60 = 3,600 seconds in 1 hour. Therefore, the speed limit can be written as 15 mi/3,600 s. Next, we'll need to convert the distance unit to feet. Knowing there are 5,280 feet in 1 mile, there are 15 × 5,280 = 79,200 feet in 15 miles. Now, we can write the speed limit as 79,200 ft/3,600 s. Since we're asked for feet per second as the final unit, we'll need to find how many feet can be traveled in 1 second. To do this, we can dividie 79,200/3,600 = 22 ft/s.

The Problems

Coach instructions: *After students try the warm-up problems, play the video and have them follow along with the solutions.*

Take a look at the following problems and follow along as they are explained in the video.

6. Lindsay starts at the peak of a mountain, and it takes her 45 minutes to hike 15,840 feet. What was her average walking speed, in miles per hour, given 1 mile = 5,280 feet?

Solution in video. Answer: 4 miles per hour.

7. Brice rides his bike at a constant speed of 8 mi/h for 15 minutes, then speeds up and rides at a constant speed of 10 mi/h for 30 minutes. During these 45 minutes, how many miles did he travel?

Solution in video. Answer: 7 miles.

8. How many minutes faster will Jacob complete a 100-mile drive traveling at a rate of 60 miles per hour than if he traveled at a rate of 50 miles per hour?

Solution in video. Answer: 20 minutes.

Piece It Together

Coach instructions: *After watching the video, give students 10 to 15 minutes to try the next five problems.*

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

9. Carmichael's favorite race car driver completes 10 laps in 10 minutes. If one lap is 2.5 miles long, what was the average **speed** of the driver, in miles per hour?

Completing 10 laps in 10 minutes is the same as completing 1 lap in 1 minute. If 1 lap is 2.5 miles long, then we know the race car driver drives 2.5 miles in 1 minute, or drives at a rate of 2.5 miles per minute. Since we're asked for the driver's speed in miles per hour, and there are 60 minutes per hour, we can say that the race car driver drives 2.5 × 60 = 150 miles in 60 minutes, or at a rate of 150 mi/h.

10. If Tom travels 135 miles in 1 hour 30 minutes, what is his **speed** in feet per second, given 1 mile = 5,280 feet?

Since we're asked to find Tom's speed, we know to use the formula rate = distance / time. We know that 1 hour 30 minutes is the same as 90 minutes, so Tom's rate is 135 mi / 90 mins. We are asked to find his rate in feet per second, so we'll need to convert these units. Knowing there are 5,280 feet per mile, 135 miles is equal to 135 × 5,280 = 712,800 feet. Knowing there are 60 seconds per minute, 90 minutes is equal to 90 × 60 = 5,400 seconds. So, Tom's rate = 712,800/5,400 = 132 ft/s.

11. A pedestrian averages 3 mi/h on the streets of Manhattan, and a subway train averages 30 mi/h. If each city block is 1/20 of a mile, how many **more minutes** than the subway train does it take for a pedestrain to travel 60 blocks in Manhattan?

Since we're asked to find a difference in time, we can focus on applying the formula time = distance / rate. First, we know that 20 blocks = 1 mile, so 60 blocks is equal to 60 ÷ 20 = 3 miles. The pedestrian travels at 3 mi/h over the 3 mile distance, so the pedestrian's walking time = 3 / 3 = 1 hour (or 60 minutes). The subway train travels at 30 mi/h over the 3 mile distance, so the train's time = 3 / 30 = 1/10 hours (or 6 minutes). So, the pedestrian takes 60 – 6 = 54 minutes more than the train to travel 60 blocks.

12. A hare is running at a rate of 1 m every minute, while a tortoise is crawling at a rate of 1 cm every second. In **meters**, how much **farther** than the tortoise will the hare travel in an hour?

Since we're asked to find the difference in distance, we can focus on applying the formula distance = rate × time. If the hare runs 1m per minute, then the hare would run 1 × 60 = 60m per hour. If the tortoise crawls 1cm per second, then the tortoise would crawl 1 × 60 = 60cm per minute. Since there are 60 minutes in 1 hour, this would equate to 60 × 60 = 3,600cm per hour. Knowing 100cm = 1m, 3,600 / 100 = 36m. So, the tortoise crawls at a rate of 36m per hour. Therefore, the hare will travel 60 – 36 = 24 meters farther than the tortoise.

13. A train traveling at 45 miles per hour enters a tunnel that is 1 mile long. The length of the train is 1/8 mile. How many **minutes after** the front of the train enters the tunnel does the back of the train exit the tunnel? Express your answer as a decimal to the nearest tenth.

From the provided information, we can determine that the front of the train must travel 1 + 1/8 = 9/8 miles from the time it enters the tunnel to the time the back of the train clears the tunnel. Since we're asked to find time, we can focus on applying the formula time = distance / rate. Knowing the train is traveling at 45 mi/h, this will take 9/8 ÷ 45 = 1/40 hours, which is equal to 1/40(60) = 1.5 minutes.

Optional Extension

***Coach instructions:** Once your students have completed the problems and feel they have a comfortable understanding of the concept, let them attempt these two more challenging rate problems.*

To extend your understanding and have a little fun with math, try the following activities.

In the problems below, you'll notice that there is not as much information provided as in the problems we've looked at so far. Even though there is "missing" information, these problems can still be solved!

If a runner who runs at a constant **speed** of p miles per hour runs a mile in exactly p minutes, what is the integer closest to the value of p?

We can use this information to set up the following equation and solve:

p **mi/h = 1 mi/**p **min × 60 min/h**

$p = 60/p$ **(canceling out units and simplyfing)**

$p^2 = 60$ **(multiplying both sides by** p**)**

$p = \sqrt{60}$ **(taking the square root of both sides)**

$p = 2\sqrt{15} \approx 2\sqrt{16} \approx 8$ **(approximating closest integer)**

Jack and Jill travel up a hill at a speed of 2 mi/h. They travel back down the hill at a speed of 4 mi/h. What is their average **speed** for the entire trip? Express your answer as a mixed number.

In this problem, we are looking for average speed. We are given two different speeds that Jack and Jill travel at. On the way up the hilll they travel at 2 mi/h and on the way down they travel at 4 mi/h. A common *incorrect* solution is averaging these two speeds outright: (2 mi/h + 4 mi/h)/2 = 3 mi/h. This might seem to make sense because typically when computing averages, such as averaging test scores, all the items have the same weight. Here this is not the case.

Think about it this way, since they traveled the same distance up the hill as they did down the hill, but they went slower going up, Jack and Jill would have been moving at a speed of 2 mi/h for longer. In fact, since the ratio of their speeds is 4:2, they would spend twice as much time traveling up hill as downhill. Knowing this, we can now calculate the average speed for thier entire trip as (2 × 2 mi/h + 4 mi/h)/3 = 8/3 mi/h = 2 2/3 mi/h.

Alternatively, since we don't know the distance, we can assign it a variable to set up an equation. Let's say the distance up the hill is d miles. On the trip up the hill, Jack and Jill would have taken d mi ÷ 2 mi/h = $d/2$ hours to get to the top. Similarly, they would have taken $d/4$ hours to travel back down the hill. The total distance traveled is $2d$ miles and the total time traveled is $d/2 + d/4 = 3d/4$ hours. The average speed is therefore $2d/(3d/4) = 8/3$ = 2 2/3 mi/h.