*This practice plan was created by **Taren Long**, a math teacher and coach at Chesapeake Public Charter School. Taren created numerous free resources for MATHCOUNTS coaches in her role as the 2020-2021 DoD STEM Ambassador for MATHCOUNTS. Find more resources and information at **dodstem.us**.*

Ratios and Area

Warm-Up!

Coach instructions: *Give students around 10 minutes (2 minutes per problem) to go through the warm-up problems.*

Try these problems before watching the lesson.

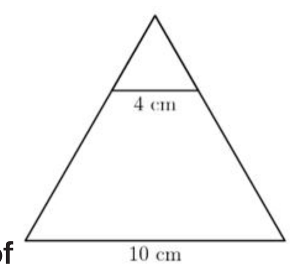
1. The ratio of the lengths of the corresponding sides of two similar decagons is 1:2. If the perimeter of the smaller decagon is 76 cm, what is the perimeter of the larger decagon?

   The ratio of the sides 1:2 is the same as the ratio of the perimeters. Thus, the larger decagon has a perimeter that is double the smaller decagon. 76 · 2 = 152 centimeters.

2. If a rectangle is formed by doubling the length of one side of a square and halving the other side of the square, what is the ratio, expressed as a common fraction, of the perimeter of the rectangle to the perimeter of the square?

   Let the sides of the square be represented by x. The square has a perimeter of $4x$. The rectangle has dimensions of $2x$ and $\frac{1}{2}x$. The perimeter can be written as $2x + 2x + \frac{1}{2}x + \frac{1}{2}x$, which simplifies to 5x. The ratio of the perimeters is 5/4.

3. In the diagram, the two triangles shown here have parallel bases. What is the ratio of the area of the smaller triangle to the area of the larger triangle? Express your answer as a common fraction.

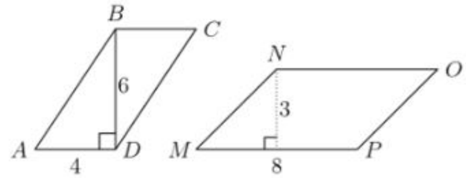


   The ratio of the areas of two figures is equal to the square of the ratio of their corresponding sides. The ratio of the sides of the triangles is 4:10. Thus, the ratio of the areas of these triangles is $4^2:10^2$ or 16:100. 16:100 simplifies to 4/25.

4. Two triangles are similar. The ratio of their areas is 1:4. If the height of the smaller triangle is 3 cm, how long is the corresponding height of the larger triangle, in centimeters?

   If the ratios of the areas of two similar triangles is 1:4, the ratio of their side lengths is $\sqrt{1}:\sqrt{4}$, or 1:2. The larger triangle has lengths that are double the smaller triangle, so if the smaller height is 3, the larger height must be 3 · 2 = 6 centimeters.

5. What is the ratio of the area of triangle ABD to the area of parallelogram MNOP, shown here? Express your answer as a common fraction.



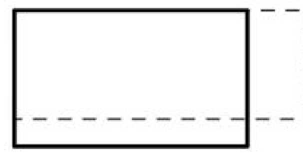
The triangle has a base of 4 and a height of 6, which gives an area of $\frac{1}{2} \cdot 4 \cdot 6 = 12 units^2$. The parallelogram has a base of 8 and a height of 3, which gives an area of $8 \cdot 3 = 24 units^2$. The ratio of the area of the triangle to the parallelogram is 12/24 or 1/2.

The Problems

***Coach instructions:** After students try the warm-up problems, play the video and have them follow along with the solutions.*

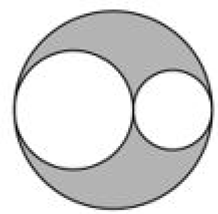
Take a look at the following problems and follow along as they are explained in the video.

6. The length of a rectangle is increased by 25%. To keep the area of the rectangle the same, by what percent must the width be decreased?



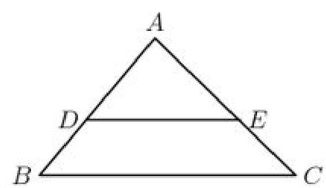
Solution in video. Answer: 20.

7. Two small circles with radii 2 cm and 3 cm are externally tangent. A third circle is circumscribed about the first two as shown. What is the ratio of the area of the smallest circle to the area of the shaded region? Express your answer as a common fraction.



Solution in video. Answer: 1/3.

8. In the figure, the area of trapezoid DBCE is 80 cm^2. The ratio of the bases DE to BC is 3:5. What is the area of triangle ADE, in square centimeters?



Solution in video. Answer: 45 square centimeters.

Piece It Together

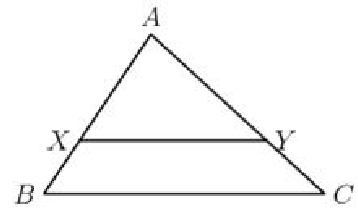
***Coach instructions:** After watching the video, give students 10 to 15 minutes to try the next four problems.*

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

9. The length of a side of a triangle with an area of 36 square inches is 4.2 inches. What is the number of square inches in the area of a similar triangle whose corresponding side measures 5.6 inches?

The ratio of the sides is 4.2:5.6. An equivalent whole-number ratio would be 42:56, which simplifies to 6:8. The ratio of the areas of these triangles is $6^2:8^2$ or 36:64. Since the smaller triangle's area is 36 square inches, the larger triangle's area is 64 square inches.

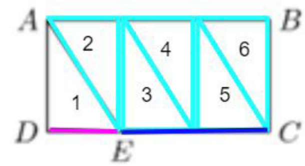
10. If AX and AY are 2/3 of AB and AC, respectively, what is the ratio of the area of triangle AXY to trapezoid XYCB? Express your answer as a common fraction.



The ratio of the areas of two figures is equal to the square of the ratio of their corresponding sides. The ratio of the areas is 2^2:3^2 or 4:9. The trapezoid XYCB is made up of the larger triangle area (9) minus the smaller triangle area (4), which is 5. Thus, the ratio of the triangle AXY to the trapezoid XYCB is 4/5.

11. AE divides rectangle ABCD into two parts such that the ratio of the area of triangle ADE to the area of the quadrilateral ABCE is 1:5. Find the ratio of DE to EC. Express your answer as a common fraction.

The area of triangle ADE is 1 unit. The quadrilateral ABCE can contain 5 triangles, which can be modeled with transformations of ADE, as shown. The ratio of the sides DE to EC is 1:2.



12. The length of a right, rectangular prism is doubled, its width is quadrupled and its height is unchanged. What is the ratio of the original volume to the new volume? Express your answer as a common fraction.

The volume of a prism is equal to length · width · height, or $l \cdot w \cdot h$. The new volume would be $(2 \cdot l) \bullet (4 \cdot w) \bullet h = 8 \cdot l \cdot w \cdot h$. **The ratio of the original volume ($l \cdot w \cdot h$) to the new volume ($8 \cdot l \cdot w \cdot h$) is 1/8.**