*This practice plan was created by **Taren Long**, a math teacher and coach at Chesapeake Public Charter School. Taren created numerous free resources for MATHCOUNTS coaches in her role as the 2020-2021 DoD STEM Ambassador for MATHCOUNTS. Find more resources and information at **dodstem.us**.*

Interior Angles of Polygons

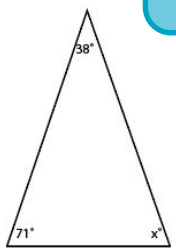
Warm-Up!

Try these problems before watching the lesson.

Coach instructions: *Give students around 7 minutes to go through the warm-up problems.*

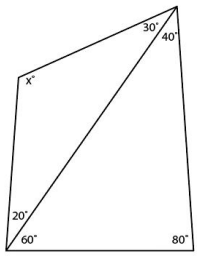
Note: *The terms in blue italics commonly appear in competition problems. Make sure Mathletes understand their meaning!*

1. The sum of the ***interior angles*** of a triangle is 180°. Solve for the missing angle in the figure at right.



The sum can be represented by the equation $180 = x + 71 + 38$.
$180 - 71 - 38 = x$. $x = 71°$.

2. What is the measure of the missing angle shown at right?



Consider using the triangles that partition the quadrilateral to derive the sum of the angles in the quadrilateral. The quadrilateral is made up of two triangles. The sum of the angles in a triangle is $180°$**, so** $180 - (20 + 30) = x$ **and** $x = 130°$.

3. What is the sum of the ***interior angles*** in a quadrilateral?

The sum of the interior angles in a quadrilateral is 360°.

4. What is the sum of the ***interior angles*** in a pentagon?

A pentagon is formed from 3 triangles, so $3 \bullet 180 = 540°$.

5. If all of the ***interior angles*** of a polygon are ***congruent***, the polygon is called a ***regular polygon***. What is the measure of each interior angle of a regular pentagon?

A pentagon has an interior sum of 540° and contains 5 angles. 540/5 = 108°.

The Problems

Coach instructions: *After students try the warm-up problems, play the video and have them follow along with the solutions.*

Take a look at the following problems and follow along as they are explained in the video.

6. If an isosceles triangle has base angles that are each twice the measure of the smaller angle, what is the measure of one of the base angles?

   Solution in video. Answer: 72°.

7. What is the measure of the sum of the ***internal angles*** of a regular dodecagon (12-sided polygon)? What is the measure of each interior angle of the regular dodecagon?

   Solution in video. Answer: 150°.

8. If the measure of an ***interior angle*** of a ***regular polygon*** is 170°, how many sides does the polygon have?

   Solution in video. Answer: 36.

Piece It Together

Coach instructions: *After watching the video, give students 10 minutes to try the next three problems.*

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

9. In the parallelogram PQRS, angle P is equal to four times angle Q. How many degrees are in the measure of angle P?

   Angle Q has measure *x*. The measure of Angle P is 4*x*. The sum of the angles of the parallelogram is $x+4x+x+4x=360°$. Combining yields $10x=360$. Dividing both sides by 10 gives the answer *x*, or angle Q = 36°. The measure of angle P is four times the measure of angle Q, so $4 \bullet 36 = 144°$.

10. By how many degrees does the measure of an ***interior angle*** of a regular decagon exceed the measure of an ***interior angle*** of a regular pentagon?

    The formula for the measure of an interior angle of a regular polygon with *n* sides is

    $\frac{180(n-2)}{n}$**. A decagon has 10 sides and a pentagon has 5 sides, so we can simplify:**

    $\frac{180(10-2)}{10}=\frac{180(8)}{10}=\frac{1440}{10}=144°$ and $\frac{180(5-2)}{5}=\frac{180(3)}{5}=\frac{540}{5}=108°$

    The difference between 144 and 108 is 144 – 108 = 36°.

11. If the measure of an ***interior angle*** of a ***regular polygon*** is 162°, and this polygon is the base of a prism, how many edges does the prism have?

We can solve the formula: $\frac{180(n-2)}{n} = 162°$. **Rearranging the equation we get** $180n - 360 = 162n$. **Solving for *n*, we get** $-360 = -18n$ **and** $n = 20$. **A 20-sided polygon has 20 edges for each of the base, sides and top. 20 + 20 + 20 = 60 edges.**

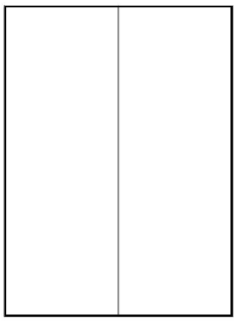
Optional Extension

Coach instructions: *Once your students have completed the problems and feel they have a comfortable understanding of the concept, let them try this paper-folding task!*

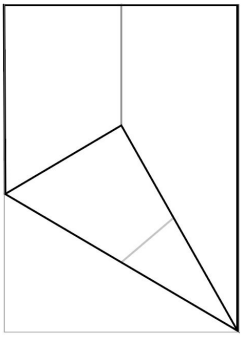
To extend your understanding and have a little fun with math, try the following activity.

Materials
- Pencil
- Piece of rectangular paper

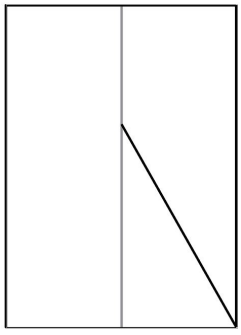
How can you make an angle of 60 degrees by folding a rectangular sheet of paper twice? Can you make an equilateral triangle with one more fold?



Take a rectangular piece of paper. Fold it along the long axis and then open it up.



Now bring the the lower left corner up and onto the center crease so that the fold line continues to the bottom right corner. You have now made a 60 degree and 30 degree angle.



Take your pencil and draw a line along the longer leg of the right triangle you folded. Re-open the paper.

Repeat the corner fold with the opposite bottom corner of the rectangle. Draw a line along the longer leg of the right triangle you formed. When you open up your paper, the two lines and the base of the paper form an equilateral triangle!