MATHCOUNTS®

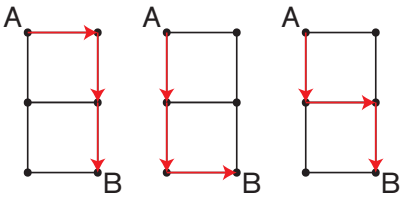
Counting Paths Along a Grid

Warm-Up!

Try these problems before watching the lesson.

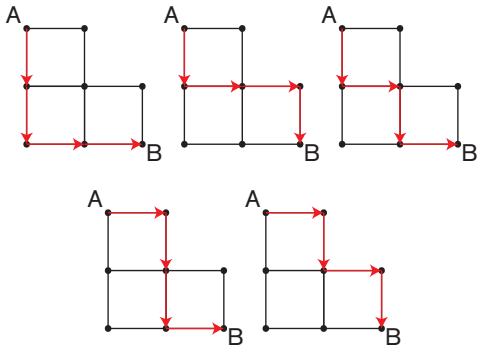
Coach instructions: *Give students around 10 minutes to go through the warm-up problems.*

1. If a ladybug walks on the segments of the diagram from point A to point B moving only to the right or downward, how many distinct paths are possible?



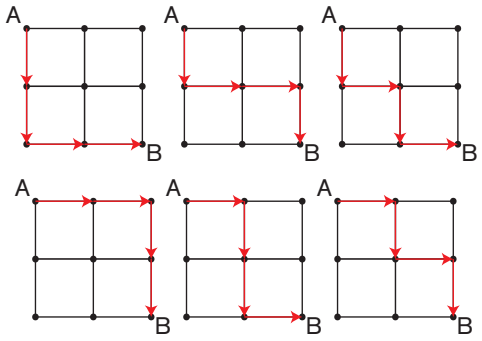
The 3 paths are shown in the figure to the right.

2. If a ladybug walks on the segments of the diagram from point A to point B moving only to the right or downward, how many distinct paths are possible?



The 5 paths are shown in the figure to the right.

3. If a ladybug walks on the segments of the diagram from point A to point B moving only to the right or downward, how many distinct paths are possible?

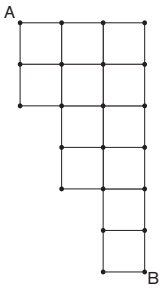


The 6 paths are shown in the figure to the right.

The Problems

Take a look at the following problems and follow along as they are explained in the video.

4. If a ladybug walks on the segments of the diagram from point A to point B moving only to the right or downward, how many distinct paths are possible?



Solution in video. Answer: 55 paths.

Coach instructions: *After Mathletes try the warm-up problems, play the video and have them follow along with the solution.*

video courtesy of

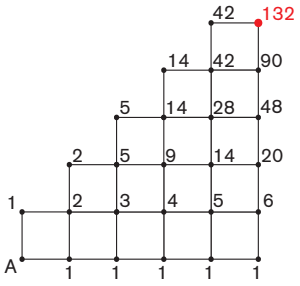
AoPS
Art of Problem Solving
www.artofproblemsolving.com

Piece It Together

Coach instructions: *After watching the video, give students 10 to 15 minutes to try the next few problems.*

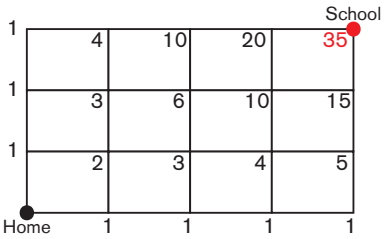
Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

5. If an ant walks on the segments of the diagram from point A to point B moving only to the right or upward, how many distinct paths are possible?



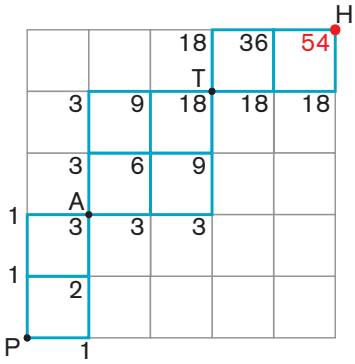
Using the counting strategy from the video, we can see there are 132 possible paths. See the figure to the right for the solution.

6. Alvin lives 4 blocks west and 3 blocks south of his school. He wants to take a different route to school each day, but each route must be exactly 7 blocks long. For how many days can he do this without repeating any route?



There are 35 possible routes to school. See the figure to the right for the solution.

7. Moving only up and right, how many paths from P to H pass through A and T?



This problem has added restrictions on the path possibilities. Because the paths must pass through A and T and can only move up and right, we do not need to consider any paths that move to the right or above the letters A and T before intersecting with them. Considering only these paths we find there is a total of 54 paths connecting P to A to T to H.

Optional Extension

Coach instructions: *Once your students have completed the problems, have them try to see the connection between counthing paths and another counting/combinatorics problem.*

To extend your understanding and have a little fun with math, try the following activities.

How many different 4-letter "words" can we form by arranging the letters M, M, C and C? Does the answer to this problem match the answer to problem 3 from the warm-up? If not, solve them both again. If they are the same, explain why these two problems are essentially answering the same mathematical question.

There are 6 ways to arrange the four letters to form "words": MMCC, MCCM, CCMM, MCMC, CMCM and CMMC. This is the same as the answer to number 3 of the warm-up problem. We can think of these two as the same problem if we assign the letters M and C to the two directions right and down in the grid. If we suppose everytime we make a move right, we add an M to our "word" and everytime we move down we add a C to our "word" then the possible paths correspond directly to the possible letter arrangements.