MATHCOUNTS® Problem of the Week Archive

Scrabble – February 5, 2024

Problems & Solutions

Here is a table showing the number of tiles on which each letter appears in a standard English Scrabble game. Note that the blank tiles can be used as any letter a player wants.

A: 9	B: 2	C: 2	D: 4	E: 12	F: 2	G: 3	H: 2	I: 9
J: 1	K: 1	L: 4	M: 2	N: 6	O: 8	P: 2	Q: 1	R: 6
S: 4	T: 6	U: 4	V: 2	W: 2	X: 1	Y: 2	Z: 1	Blank: 2

Jennifer and Pete are going to play Scrabble with their brand new Scrabble game. They have decided that in order to determine who will go first, they will each randomly select one letter, without replacement, from the bag containing all of the letter tiles except the 2 blank tiles. The person who selects a letter closest to A goes first (if they both draw the same letter they will draw again). Jennifer draws first and selects an E. What is the probability that Pete's selection will result in Pete going first (without having to redraw)?

*There are 98 tiles that have letters printed on them, so after Jennifer selects her letter, Pete is left with 97 tiles to draw from. In order for Pete to go first, he must select an A, B, C or D. There are 9 + 2 + 2 + 4 = 17 such tiles. Thus, the probability that Pete will go first is **17/97**.*

After determining who goes first, Jennifer and Pete each put their letter back in the bag, added the 2 blank tiles, and then shook the bag to mix the letters. Now each person will draw seven letters (drawing one letter at a time) to be used on their first turn. Pete selects his 7 letters first and ends up with 2 Es, 1 R, 1 H, 1 S, 1 K and 1 O. Now it is Jennifer's turn to select 7 letters. What is the probability that the first letter Jennifer selects will be a vowel (A, E, I, O or U)? Express your answer as a common fraction.

*There are 100 tiles in the bag before anyone makes a selection, thus after Pete selects his 7 tiles, there are 93 tiles from which Jennifer gets to draw. Also, notice that there are 9 + 12 + 9 + 8 + 4 = 42 vowels when all of the tiles are in the bag, but since Pete drew 2 + 1 = 3 vowels, there are 42 – 3 = 39 vowels left in the bag when Jennifer goes to make her first draw. Thus, the probability of Jennifer drawing a vowel on her first draw is 39/93 = **13/31**.*

Jennifer ended up drawing 1 A, 2 Is, 1 S, 1 D, 1 U and 1 N. If the 2 I tiles are indistinguishable, in how many distinct orders can the 7 tiles be placed on Jennifer's letter tray?

*If each of the letter tiles were distinct, the number of orders would be 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040, since there would be seven distinct options for the first spot, leaving 6 distinct options for the second spot, and so on. However, in this case, there are 2 tiles that are the same. We can handle this issue by dividing 7! by 2, because half of the orders counted in 7! are due to the 2 identical Is swapping places. Thus, there are 7! ÷ 2 = **2520** distinct orders.*

MATHCOUNTS® Problem of the Week Archive

Scrabble – February 5, 2024

Problems

Here is a table showing the number of tiles on which each letter appears in a standard English Scrabble game. Note that the blank tiles can be used as any letter a player wants.

A: 9	B: 2	C: 2	D: 4	E: 12	F: 2	G: 3	H: 2	I: 9
J: 1	K: 1	L: 4	M: 2	N: 6	O: 8	P: 2	Q: 1	R: 6
S: 4	T: 6	U: 4	V: 2	W: 2	X: 1	Y: 2	Z: 1	Blank: 2

Jennifer and Pete are going to play Scrabble with their brand new Scrabble game. They have decided that in order to determine who will go first, they will each randomly select one letter, without replacement, from the bag containing all of the letter tiles except the 2 blank tiles. The person who selects a letter closest to A goes first (if they both draw the same letter they will draw again). Jennifer draws first and selects an E. What is the probability that Pete's selection will result in Pete going first (without having to redraw)?

After determining who goes first, Jennifer and Pete each put their letter back in the bag, added the 2 blank tiles, and then shook the bag to mix the letters. Now each person will draw seven letters (drawing one letter at a time) to be used on their first turn. Pete selects his 7 letters first and ends up with 2 Es, 1 R, 1 H, 1 S, 1 K and 1 O. Now it is Jennifer's turn to select 7 letters. What is the probability that the first letter Jennifer selects will be a vowel (A, E, I, O or U)? Express your answer as a common fraction.

Jennifer ended up drawing 1 A, 2 Is, 1 S, 1 D, 1 U and 1 N. If the 2 I tiles are indistinguishable, in how many distinct orders can the 7 tiles be placed on Jennifer's letter tray?